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Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

Practice Paper 1

Paper Reference
8FM0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 80. There are 10 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations:

$$l_1\text{: } \mathbf{r} = (-3\mathbf{i} + 5\mathbf{k}) + \lambda(5\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$l_2\text{: } \mathbf{r} = (10\mathbf{i} - \mathbf{j} + 15\mathbf{k}) + \mu(6\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$$

Show that lines l_1 and l_2 do not meet.

(Total for Question 1 is 4 marks)

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2. $\mathbf{M} = \begin{pmatrix} 2 & k & 3 \\ 1 & -3 & 1 \\ 3 & -1 & 2 \end{pmatrix}$ where k is an integer.

(a) Find det $\mathbf{M}$, giving your answer in terms of k.

(3)

Three planes A, B and C are defined by the following Cartesian equations:

A: $2x + ky + 3z = 1$

B: $x - 3y + z = -2$

C: $3x - y + 2z = 3$

(b) Given that the planes do not meet at a single point, determine whether the three equations form a consistent system, and give a geometric interpretation of your answer. You must show sufficient working to justify your conclusions.

(5)

(Total for Question 2 is 7 marks)

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3. The cubic equation $2x^3 - 3x^2 - 7x - 1 = 0$ has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(2\alpha - 1)$, $(2\beta - 1)$ and $(2\gamma - 1)$, giving your answer in the form $pw^3 + qw^2 + rw + s = 0$ where p, q, r and s are integers to be found.

(Total for Question 3 is 6 marks)

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4. (a) Prove by induction that for all positive integers n,

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(6)

(b) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} 2r(r+1) = \frac{2}{3}n(n+1)(n+2)$$

(4)

(c) Hence show that $n = 1$ is the only value of n that satisfies

$$\sum_{r=1}^{n} 2r(r+1) = 4\sum_{r=1}^{n} r^3$$

(5)

(Total for Question 4 is 15 marks)

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5. $$\mathrm{f}(z) = z^4 - 14z^3 + 78z^2 + kz + 221, \text{ where } k \text{ is a real constant.}$$

Given that $z = 3 - 2\mathrm{i}$ is a root of the equation $\mathrm{f}(z) = 0$,

(a) show that $z^2 - 6z + 13$ is a factor of $\mathrm{f}(z)$

(4)

(b) find the value of k

(1)

(c) solve completely the equation $\mathrm{f}(z) = 0$ and show the roots on an Argand diagram.

(4)

(Total for Question 5 is 9 marks)

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6. The plane Π has Cartesian equation $x - y + 2z = 3$.

(a) Find a unit vector $\hat{\mathbf{n}}$ normal to Π.

(2)

A line l has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix}$.

The line intersects Π at point P.

(b) Find the coordinates of P and the acute angle between l and Π, giving your answer in radians correct to two decimal places.

(7)

(Total for Question 6 is 9 marks)

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7. Frances makes silver jewellery beads. Each bead is formed by rotating the curve shown in Figure 1 through 2π radians about the x-axis.

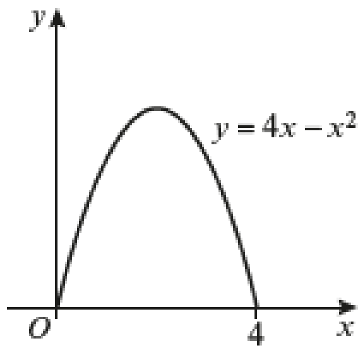

Figure 1

Each unit on the axes represents 1 mm. Silver costs £0.05 per cubic millimetre.

(a) Find the cost of the silver needed to make 500 beads.

(6)

(b) State one limitation of this model.

(1)

(Total for Question 7 is 7 marks)

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8. The matrix $\mathbf{M} = \begin{pmatrix} -\dfrac{5}{\sqrt{2}} & -\dfrac{5}{\sqrt{2}} \\ \dfrac{5}{\sqrt{2}} & -\dfrac{5}{\sqrt{2}} \end{pmatrix}$ represents an enlargement followed by a rotation.

(a) Find the scale factor of the enlargement.

(2)

(b) Find the angle of rotation.

(2)

A point P is mapped onto a point P' under $\mathbf{M}$.

Given that the coordinates of P' are (a, b),

(c) find, in terms of a and b, the coordinates of P.

(4)

(Total for Question 8 is 8 marks)

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9. Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 2\mathrm{i}| \leq 3\} \cap \{z \in \mathbb{C} : \frac{\pi}{2} < \arg(z - 2 + \mathrm{i}) \leq \frac{3\pi}{4}\}.$$

(Total for Question 9 is 6 marks)

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10. A stolen car is modelled as travelling in a straight line from a point $A(5, 3, -1)$ to a point $B(7, -5, 8)$. The police are waiting for the car to pass at a point modelled as the origin, O. The police have a 'stinger' that they can deploy to stop the car. The 'stinger' is 6 metres in length.

(a) Given that each unit in the model represents one metre, determine whether or not the police can successfully stop the car.

(7)

(b) State one limitation of this model.

(1)

(Total for Question 10 is 8 marks)

TOTAL FOR CORE PURE MATHEMATICS IS 80 MARKS

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